PROBLEM OF A PISTON IN A PLASTIC MEDIUM WITH CULTIVATION

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The article discusses the self-similar problem of the motion of a spherical piston in a medium with "dry" friction and the differential connection between the first invariant of the stress tensor and the second invariant of the tensor of the deformation rates. For the case of flow with a shock wave, the method of a stationary wave is used to obtain the required relationships at a strong discontinuity. It is disclosed that the velocity of the piston, as well as the relationship between the cultivation coefficients and the dry friction, are determined by the smoothness of the friction.

Article [1] describes models of loose soils, in which the second invariant of the deviator of the stress tensor T_2 is connected with the pressure by the relationship

$$\sqrt{T_2} = \pm \varkappa p \tag{1}$$

where $\varkappa$ is the coefficient of dry friction, and the sign is selected depending on the sign of the second invariant of the deviator of the tensor of the deformation rates T_2. Such models are partial cases of models of plastic bodies and are called models of media with dry friction. The condition of a hyperbolic character of the system of equations of motion imposes limitations on the possible values of the coefficient of dry friction $|\varkappa| < 3/4$.

To determine the pressure, we must introduce into the hydrodynamics an additional relationship, analogous to the relationship $p_e = f(\rho)$. With the motion of a soil, in distinction from the motion of liquids, a change in the pressure may take place not only due to a change in the density, but also as a result of a rise in the shear deformations. Similar models are proposed in [2-4]. In these models, the increment of the pressure is represented in the form of the sum of two terms, the first of which reflects the hydrostatic compression, and the second, the change in the pressure due to a change in the shear deformations

$$dp = dp_h + dp_\alpha, \quad dp_h = Fd\rho \tag{2}$$
$$dp_\alpha = F\rho q\Psi(p, \rho)\sqrt{I_2}dt$$

The cultivation function $\Psi(p, \rho)$ must satisfy a number of conditions: 1) the function $\Psi(p, \rho)$ must be of constant sign; 2) the system of equations of motion and the equations of state (1), (2) must have real characteristics. Since the presence of real characteristics is connected with the existence of a finite rate of propagation of small perturbations in the medium, which is equal to

$$a^2 = F(1 \pm q\Psi(p, \rho))\left(1 \pm \frac{4}{3}\varkappa\right) \tag{3}$$

then, the cultivation function must satisfy the inequality

$$1 \pm q\Psi(p, \rho) > 0$$

The cultivation function can be chosen by many methods.

Specifically, it can be represented in two forms:

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$$\Psi(p, \rho) = \Psi_0 \equiv \text{const}$$
$$\Psi(p,\rho) = 1 - \left(\rho^* + \frac{1}{F}p\right)\rho^{-1}, \quad \rho_0 > \rho^*$$

To bring out the effect of cultivation on the form of the shock adiabatic curve, on the profiles of the velocities and the pressures, and on the qualitative picture of the flow, we shall use an arbitrary way of writing the function $\Psi(p, \rho)$. As an illustration of the results, we take the cultivation function $\Psi(p, \rho)$ in the form $\Psi = \Psi_0$. Such a choice of the function $\Psi(p, \rho)$ does not limit the results obtained, but considerably simplifies the proof and the computations. Cases where the character of the cultivation function affects the conclusions will be stipulated specially.

The equations of motion and the equations of state in a spherical system of coordinates assume the form

$$\begin{aligned}
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = \frac{1}{\rho}\frac{\partial \sigma^r}{\partial r} + \frac{2(\sigma^r - \sigma^\theta)}{\rho r} \\
&\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2u\rho}{r} = 0 \\
&\sqrt{T_2} = \frac{\sigma^r - \sigma^\theta}{2} = \pm \varkappa p, \quad p \geqslant 0 \\
&\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} = F\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial t} + q\Psi(p,\rho)\rho\left|\frac{\partial u}{\partial r} - \frac{u}{r}\right|\right) \\
&\sqrt{I_2} = \left|\frac{\partial u}{\partial r} - \frac{u}{r}\right|
\end{aligned} \tag{4}$$

The equations of motion of a friable medium with the equation of state (4) must be satisfied by the relationships at the front of the shock wave. While the integrals of the conservation of mass and momentum immediately give two of the required relationships, due to the nonholonomic character of Eq. (2), the third equation at the front of the wave can be obtained by the examination of the structure of a plane stationary wave, propagating along a friable viscous medium, under the assumption that inside a transitional layer, simulating a shock wave, the previous equations of state are retained. The structure of shock waves for different equations of state of the soil are discussed in [5, 6] by similar methods.

The equations of motion of a plane one-dimensional flow of a viscous friable medium, in Lagrangian coordinates, have the form

$$\begin{aligned}
&\frac{\partial V}{\partial t} = V_0\frac{\partial u}{\partial x}, \qquad \frac{\partial u}{\partial t} = V_0\frac{\partial(\sigma^x + S)}{\partial x} \\
&\sigma^x = -p + \sqrt{T_2}, \qquad p = -{}^1/_3(\sigma^x + 2\sigma^y) \\
&\sqrt{T_2} = \frac{\sigma^x - \sigma^y}{2} = \varkappa p \\
&\frac{\partial p}{\partial t} = \frac{F}{V^2}\left(-\frac{\partial V}{\partial t} + q\Psi(p,\rho)\left|\frac{\partial u}{\partial x}\right|V_0\right) \\
&S = -\nu\frac{V_0}{V}\frac{\partial u}{\partial x}
\end{aligned} \tag{5}$$

Here V is the specific volume; ρ is the density; u is the velocity; σ^x and σ^y is the stress; p is the pressure.

Considering a steady-state flow, reflecting the motion of a shock wave with the velocity c along a quiescent medium with $\rho = \rho_0$ and $p_0 = 0$, introducing the variable $w = x - ct$, we transform system (5) to the form

$$\begin{aligned}
&-c\frac{dV}{dw} = V_0\frac{du}{dw} \\
&-c\frac{du}{dw} = -V_0\left(1 - \frac{4}{3}\varkappa\right)\frac{dp}{dw} - V_0\frac{dS}{dw} \\
&-c\frac{dp}{dw} = \frac{F}{V^2}\left(c\frac{dV}{dw} - q\Psi(p,\rho)c\beta\frac{dV}{dw}\right) \\
&\beta = \operatorname{sign}\left(\frac{\partial V}{\partial t}\right) = \operatorname{sign}\left(\frac{\partial u}{\partial x}\right)
\end{aligned} \tag{6}$$

Let $M = V_0^{-1}c$ be the flow of mass through an arbitrary plane; integrating the first two equations in (6), we obtain

$$MV + u = MV_0 - c, \quad Mu - (1 - {}^4/_3\varkappa)p - S = -Mc$$

The third relationship is the differential equation,

$$\frac{dp}{dV} = -\frac{F}{V^2}(1 - q\beta\Psi(p,\rho)) \tag{7}$$

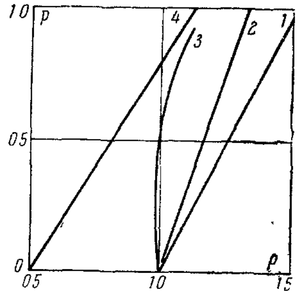

Fig. 1

Integrating this equation gives a third relationship, analogous to the Hugoniot relationship for isothermal gases, and connecting the pressure and the specific volume at the front of the wave $p = f(V)$. The form of the function $f(V)$ depends essentially on the form of the cultivation function $\Psi(p, \rho)$. Equation (7) is integrated taking account of the sign of the derivative $\partial V/\partial t$ at the front of the wave. For $\Psi = \Psi_0$ it can be shown that the value of V decreases monotonically, $\beta = -1$, and Eq. (7) assumes the form

$$\frac{dp}{dV} = -\frac{F}{V^2}(1 + q\Psi_0) \tag{8}$$

It can be shown that for the cultivation function $\Psi(p, \rho) = 1 - (\rho^* + F^{-1}p)\rho^{-1}$ the conclusion with respect to the monotonicity of the density at the front of the wave is valid.

The form of relationship (8) does not depend on the form of the viscosity introduced S.

The structure of the shock wave is determined by solution of the equation

$$M^2(V_0 - V) - (1 - {}^4/_3\varkappa)p - S = 0$$
$$M^2(V_0 - V) - \left(1 - \frac{4}{3}\varkappa\right)f(V) + \nu\frac{c}{V}\frac{dV}{dw} = 0$$

With $w \to \pm\infty$, $dV/dw \to 0$, and the relationship connecting the velocity of the shock wave with the specific density at the front of the wave assumes the form

$$M^2(V_0 - V) = (1 - {}^4/_3\varkappa) f(V) \tag{9}$$

For the selected functions $\psi(p, \rho) = \psi_0$ and $F = \text{const}$, the function $f(V)$ has the form

$$f(V) = F(1 + q\Psi_0)\left(\frac{1}{V} - \frac{1}{V_0}\right) \tag{10}$$

The derivative $f'' > 0$ and Eq. (9) have two roots, under the condition that

$$M^2 > \frac{F}{V_0^2}(1 + q\Psi_0)\left(1 - \frac{4}{3}\varkappa\right) = \frac{a_0^2}{V_0^2}$$
$$M^2 < \frac{F}{V_1^2}(1 + q\Psi_0)\left(1 - \frac{4}{3}\varkappa\right) = \frac{a_1^2}{V_1^2}$$
$$c^2 > a_0^2, \quad c^2 < a_1^2$$

Subscript 1 relates to the state behind the front of the wave. Thus, the shock wave must move with respect to the mass ahead of the front with a supersonic velocity, and, with respect to the mass behind the front, at a subsonic velocity.

For $\Psi(p, \rho) = 1 - (\rho^* + F^{-1}p)\rho^{-1}$, integration of Eq. (7) gives

$$p = F\left[(\rho - \rho^*) - \left(\frac{\rho_0}{\rho}\right)^q(\rho_0 - \rho^*)\right] \tag{11}$$

With $q = 0$, this relationship, like relationship (10), gives the condition $p = F(\rho - \rho_0)$ for nonfriable media; with $q \neq 0$, with large compressions, Eq. (11) assumes the form

$$p = F(\rho - \rho^*)$$

For this cultivation function, the conclusion with respect to the relationship between the velocity of the shock wave and the velocities of sound before and after the shock wave, for $\Psi(p, \rho) = \Psi_0$, is also valid.

Figure 1 gives curves of $p = f(\rho)$ for nonfriable media (1), for a medium with the cultivation function $\Psi = \Psi_0$ (2), and for a medium with the cultivation function $\Psi(p, \rho) = [1 - (\rho^* + p/F)\rho^{-1}]$ (3). The straight line 4 corresponds to the equation $p = F(\rho = \rho^*)$.

Let us consider the problem of the motion of a spherical piston in a friable medium, described by the equations of state

$$\frac{dp}{dt} = F\left(\frac{d\rho}{dt} - q\rho\Psi(p, \rho)\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right)\right)$$
$$\frac{\sigma^r - \sigma^\theta}{2} = \varkappa p, \quad \varkappa < 0$$

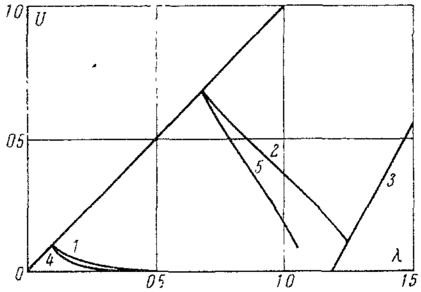

Fig. 2

The piston starts its movement from the origin of coordinates and moves with the constant velocity u_p. The unperturbed medium is characterized by a density $\rho = \rho_0$, a pressure $p = 0$, and a velocity $u = 0$. Analysis of the dimensional parameters determining the problem shows that the motion of the piston is self-similar.

Introducing the self-similar variables

$$\lambda = rt^{-1}\left(\left(1-\frac{4}{3}\varkappa\right)F\right)^{-1/2}, \quad U = u\left(\left(1-\frac{4}{3}\varkappa\right)F\right)^{-1/2}$$

$$R = \rho\rho_0^{-1}, \quad P = p\rho_0^{-1}F^{-1}, \quad \alpha = 4\varkappa\left(1-\frac{4}{3}\varkappa\right), \quad -\frac{3}{2} \leqslant \alpha \leqslant 0$$

and transforming the starting equations of motion and state, we obtain

$$\frac{dU}{d\lambda}(U-\lambda) + \frac{1}{R}\frac{dP}{d\lambda} = \frac{\alpha P}{R\lambda}, \quad \frac{dR}{d\lambda}(U-\lambda) + R\frac{du}{d\lambda} = -\frac{2UR}{\lambda},$$
$$\frac{dP}{d\lambda}(U-\lambda) + R(1+q\Psi(P,R))\frac{dU}{d\lambda} = -(2-q\Psi(P,R))\frac{UR}{\lambda} \tag{12}$$

The boundary conditions for the system (12), corresponding to a constant velocity of the piston with $\lambda = u_p[(1-\frac{4}{3}\varkappa)F]^{-1/2}$, and to a state of rest at infinity, have the form: $U = \lambda$ at the piston, $U \to 0$, $P \to 0$, $R \to 1$ with $\lambda \to \infty$. Solving system (12) with respect to the derivatives $dU/d\lambda$, $dP/d\lambda$, $dR/d\lambda$, we obtain

$$\frac{dU}{d\lambda} = \frac{\Delta_1}{(U-\lambda)\Delta}, \quad \frac{dP}{d\lambda} = \frac{\Delta_2}{(U-\lambda)\Delta}, \quad \frac{dR}{d\lambda} = \frac{\Delta_3}{(U-\lambda)\Delta}$$
$$\Delta_1 = (U-\lambda)[(U-\lambda)\alpha PR^{-1} + U(2-q\Psi(P,R))]\lambda^{-1}$$
$$\Delta_2 = -(U-\lambda)[(U-\lambda)UR(2-q\Psi(P,R)) + \alpha P(1+q\Psi(P,R))]\lambda^{-1} \tag{13}$$
$$\Delta_3 = -[2(U-\lambda)^2 UR - 3URq\Psi(P,R) + (U-\lambda)\alpha P]\lambda^{-1}$$
$$\Delta = (U-\lambda)^2 - (1+q\Psi(P,R))$$

In accordance with formula (3), the dimensionless velocity of the propagation of small perturbations is determined by the expression

$$a^2 = R(1 \pm q\Psi(P,R))$$

To Eqs. (12) there must be added the conditions at a strong discontinuity. In self-similar variables these conditions assume the form

$$R(U-\lambda) = -\lambda, \quad P + RU(U-\lambda) = 0, \quad P = f(R) \tag{14}$$

For the previously chosen cultivation functions, the third relationship in (14) assumes the form, respectively,

$$P = (R-1)(1+\Psi_0) \tag{15}$$

$$P = (R-R^*) - \left(\frac{1}{R}\right)^q (1-R^*), \quad R^* = \rho^*\rho_0^{-1} \tag{16}$$

The system of algebraic equations for determining the singular points of system (12) has the form

$$\Delta_1 = 0, \quad \Delta_2 = 0, \quad \Delta_3 = 0, \quad (U-\lambda)\Delta = 0 \tag{17}$$

The condition $U - \lambda = 0$ gives the singular points, determined by the system

$$U = \lambda, \quad UR\Psi(P,R) = 0$$

Let $U \neq \lambda$. In this case, multiplying the first equation by $-\alpha P$, the second by $U - \lambda$, and combining them, we obtain the third equation from (17). Analogously, we obtain the result that the fourth equation in (17) is a consequence of the first two. Thus, with $U \neq \lambda$, the system for determining the singular points assumes the form

$$(U-\lambda)^2 - 1 - q\Psi(P,R) = 0$$
$$\alpha(U-\lambda)P + UR(2-q\Psi(P,R)) = 0$$

Let us consider the behavior of the integral curves of the system in the neighborhood of the singular point $U = 0$, $P = 0$, $R = 1$, $\lambda = \lambda_C = [1 + q\Psi(0,1)]^{1/2}$. In accordance with [7], we consider a system of linear differential equations, obtained from system (12). Defining $S = R - 1$, $\delta = \lambda - \lambda_C$, we obtain

$$\frac{dU}{d\tau} = -AU + \alpha\lambda_C P, \quad \frac{dP}{d\tau} = -\lambda_C AU + \alpha\lambda_C^2 P$$
$$\frac{dS}{d\tau} = -\frac{A}{\lambda_C^2}U + \alpha P, \quad \frac{d\delta}{d\tau} = 2\lambda_C^2 U - \lambda_C BS - 2\lambda_C^2\delta - \lambda_C DP$$
$$A = 2 - q\Psi(0, 1), \quad B = -q\frac{\partial\Psi(0, 1)}{\partial R}, \quad D = -q\frac{\partial\Psi(0, 1)}{\partial P}$$

The first two equations can be solved independently of the remaining ones:

$$U = C_1 + C_2 \exp(\xi\tau), \quad P = C_1\frac{A}{\alpha\lambda_C} + C_2\lambda_C \exp(\xi\tau)$$

where ξ is a root of the characteristic equation $\xi^2 + (A - \alpha\lambda_C^2)\,\xi = 0$, $\xi = \alpha\lambda_C^2 - A$. To consider trajectories which enter into a singular point, we set $C_1 = 0$. Since $\xi < 0$, the trajectories enter into a singular point with $\tau \to \infty$.

Substituting the expressions obtained for U and P into the remaining equations, integrating them, and discarding solutions which do not pass through a singular point, we obtain (for $\xi + 2\lambda_C^2 \neq 0$)

$$U = C_2 \exp(\xi\tau), \quad P = C_2\lambda_C \exp(\xi\tau), \quad S = C_2\lambda_C^{-1}\exp(\xi\tau)$$
$$\delta = C_2(\xi + 2\lambda_C^2)^{-1}((2 - D)\lambda_C^2 - B)\exp(\xi\tau) + C_4\exp(-2\lambda_C^2\tau)$$

The character of the integral curves depends on the sign of the quantity $\xi + 2\lambda_C^2$.

Let $\xi + 2\lambda_C^2 < 0$. We have

$$\exp(\xi\tau) = C_2^{-1}U$$
$$\delta = ((2 - D)\lambda_C^2 - B)(\xi + 2\lambda_C^2)^{-1}U + C_4(UC_2^{-1})^{-2\lambda_C^2/\xi}$$

Since $2\lambda_C^2 + \xi < 0$, then with $C_4 \neq 0$ $(d\delta / dU)|_{u\to 0} \to \infty$. This means that all the trajectories, with the exception of one, enter a singular point with a zero slope. The solution with $C_4 = 0$ determines the second separatrix, entering the singular point with the slope

$$\frac{d\delta}{dU} = \frac{(2 - D)\lambda_C^2 - B}{\xi + 2\lambda_C^2}$$

Such a behavior of the trajectories in the neighborhood of the singular point is also characteristic for friable media without cultivation [8]. The values $C_2 > 0$ give trajectories at which $U > 0$, $P > 0$, $R > 0$, i.e., they correspond to physically possible states.

We shall show that trajectories which are a continuation of these solutions in the neighborhood of the singular point $\lambda = \lambda_C$ are physically possible, i.e., along them $U > 0$, $P > 0$, $R > 0$ up to intersection with the straight line $U = \lambda$. Thus, we shall demonstrate the assertion that the motion of a piston in a friable medium with cultivation can take place without the formation of shock waves or of weak discontinuities.

We take the cultivation function in the form $\Psi(P, R) = \Psi_0$. In this case, since $dU/d\lambda < 0$, $dP/d\lambda < 0$ in the neighborhood of the singular point $\lambda = \lambda_C$, then $\Delta < 0$ and the signs of the derivatives $du/d\lambda$ and $dP/d\lambda$ are determined by the signs of Δ_1 and Δ_2. If $U > 0$, $P > 0$, then $\Delta_1 < 0$, $\Delta_2 < 0$ and, consequently, along all the trajectories, $dU/d\lambda < 0$, $dP/d\lambda < 0$. This means that the values of the velocity and the pressure rise monotonically from the front of the perturbation toward the piston. We shall show that $R > 0$. In the neighborhood of a singular point, $dR/d\lambda < 0$, $R > 0$ and, consequently, if R reverts to zero at the point $\lambda = \lambda^*$, then $(dR/d\lambda)|_{\lambda=\lambda^*} > 0$. But from (13) it follows that

$$dR/d\lambda = -\alpha P/\lambda^*\Delta < 0$$

i.e., R does not revert to zero.

The profile of R is essentially nonmonotonic. This follows from the fact that $dR/d\lambda < 0$ in the neighborhood of a singular point, while, with motion toward the piston, $dR/d\lambda \to +\infty$.

The conclusion with respect to the existence of continuous solutions, the monotonic character of the profiles of U and P, and the nonmonotonic character of the profile of R, is valid also for a cultivation function of the form $\Psi = 1 - (\rho^* + p/F)\rho^{-1}$ and $\Psi = 1 - \rho^*\rho^{-1}$.

The solution entering a singular point with the slope $[(2-D)\lambda_C^2 - B]\cdot(\xi + 2\lambda_C^2)^{-1}$ corresponds to flow conditions with a weak discontinuity, propagating ahead of the piston, and, with its intersection with the straight line $U = \lambda$, determines the limiting velocity of the piston U_p^*, up to which continuous flow condi-

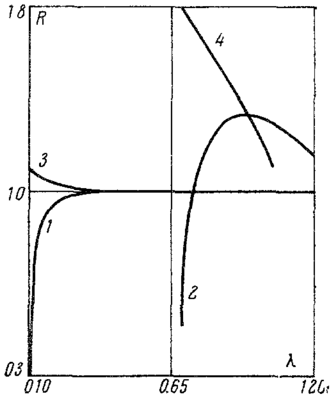

Fig. 3

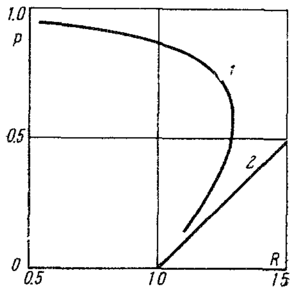

Fig. 4

tions are possible. With $U_p > U_p^*$, the only physically possible trajectory is a trajectory connecting the point $U = \lambda$ with some point on the shock wave.

We shall show that along such trajectories $\Delta < 0$ and, consequently, the profiles of U and P rise monotonically with motion away from the shock wave toward the piston. The determinant Δ at the piston is the difference between two velocities: the velocity of the shock wave with respect to the medium behind the front and the velocity of sound behind the front $1 + q\Psi(0, 1)$. It has been shown earlier that this difference is negative. The values of Δ_1 and Δ_2 are positive and, consequently, analogously to the case of continuous flow, the profiles of U and P are monotonic.

Let us consider the profile of the density. It follows from (14) that at the shock wave

$$\lambda = \sqrt{Rf(R)/(R-1)}, \qquad U = \sqrt{(R-1)f(R)/R}$$

$$\Delta_3 = -\frac{1}{R}[(2-\alpha)f(R) - 3q\Psi(P, R)(R-1)R]$$

For $q\Psi(P, R) = \Psi_0$ and $f(R) = (1+\Psi_0)(R-1)$, we obtain

$$\Delta_3 = -\frac{R-1}{R}[(2-\alpha)(1+\Psi_0) - 3\Psi_0 R]$$

With $R > R^* = {}^1/_3(2-\alpha)(1+\Psi_0)\Psi_0^{-1}$, behind the front of the shock wave, the density starts to fall. For shock waves of sufficiently great intensity, the process of cultivation is so considerable that it leads to a drop in the density immediately behind the front of the shock wave. Let us evaluate the lower boundary of the change in R*. Substituting the limiting values of α and $\Psi_0(\alpha_0 = 0,\ \Psi_0 = 1)$ we find that in this case $R^* > {}^4/_3$.

For a cultivation function of the type $1 - (\rho^* + p/F)\rho^{-1}$, it can be shown that behind the front of a shock wave the density always rises, i.e., behind the front of a shock wave $\Delta_3 < 0$.

Figure 2 gives profiles of the velocity U (curves 1 and 2), and of the shock adiabatic curve (curve 3) for the case of continuous and shock conditions, with a cultivation function $\Psi = \Psi_0 = 0.4$ and a value of $\alpha = -1.2$. The same figure gives velocity profiles (curves 4 and 5) for the same values of U_p and α in a medium without cultivation ($\Psi_0 = 0$).

Figure 3 gives characteristic profiles of the density with $\Psi_0 = 0.4$, $\alpha = -1.2$ (curves 1 and 2), as well as the corresponding densities for a medium without cultivation (curves 3 and 4).

In Fig. 4, curve 1 corresponds to the change in the pressure as a function of the density in a moving particle of a substance with cultivation and curve 2, to the change in the pressure with the density in a medium without cultivation.

Let us consider the case $\xi + 2\lambda_C^2 > 0$. The solution of the system of equations in the neighborhood of the singular point $\lambda = \lambda_C$ determines $\delta(U)$ in the form

$$\delta = ((2-D)\lambda_C^2 - B)(\xi + 2\lambda_C^2)^{-1}U + C_4(UC_2^{-1})^{-2\lambda_C^2/\xi}$$

$$\frac{d\delta}{dU} = ((2-D)\lambda_C^2 - B)(\xi + 2\lambda_C^2)^{-1} - \frac{2\lambda_C}{\xi}C_4C_2^{-2\lambda_C/\xi}U^{-(2\lambda_C^2+\xi)/\xi}$$

Since $2\lambda_C^2 + \xi > 0$, then with $C_2 \neq 0$,

$$\lim_{\lambda \to \lambda_C} \frac{d\delta}{dU} = ((2-D)\lambda_C^2 - B)(\xi + 2\lambda_C^2)^{-1}$$

For the cultivation function $\Psi(P, R) = \Psi_0$, $\lim d\delta/dU > 0$. This means that almost all the trajectories enter a singular point with the positive slope $2\lambda_C^2(\xi + 2\lambda_C^2)^{-1}$ and, consequently, motion of the medium cannot take place without a shock wave. For cultivation functions of other kinds, the question of the possibility of flow without a shock wave must be resolved anew in each case.

In this case, the character of the flow does not differ qualitatively from the shock flow conditions under consideration.

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